

# Possible scenario for MaVaN's as the only neutrino flavor conversion mechanism in the Sun

P. C. de Holanda\*

*Instituto de Física "Gleb Wataghin"*

*Universidade Estadual de Campinas, UNICAMP*

*13083-970, Campinas, São Paulo, Brasil*

## Abstract

Mass Varying neutrino mechanisms were proposed to link the neutrino mass scale with dark energy, addressing the coincidence problem. In some scenarios this mass can present a dependence on the baryonic density felt by neutrinos, creating an effective neutrino mass that depends both on the neutrino and baryonic densities. In this article we investigate the possibility that a neutrino effective mass is the only flavour conversion mechanism acting in neutrino oscillation experiments. We present a parameterization on the environmental effects on neutrino mass that produces the right flavour conversion probabilities for solar and terrestrial neutrinos experiments.

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\*Electronic address: holanda@ifi.unicamp.br

## I. INTRODUCTION

The proposal that Mass Varying Neutrinos are coupled to dark energy [1] results in a fluid with negative pressure that could mimic the effects of a cosmological constant and induce a cosmic acceleration. The cosmological consequences of such coupling were widely addressed in a number of papers [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

Effects on neutrino oscillation [19, 20, 21, 22, 23, 24, 25, 26, 27, 28] were analyzed in different scenarios. In particular, it was proposed that the Mass Varying Neutrinos phenomenology could lead to a neutrino mass dependence with baryonic density [19] through non-renormalizable operators that couple the acceleration to the baryonic matter. In ref. [23] some limits on the product of the effective neutrino-scalar and matter-scalar Yukawa couplings were obtained by comparing the solar neutrino and KamLAND data, assuming that MaVaN's mechanism plays a sub-leading role in neutrino flavour conversion.

We present in this article a possible scenario where the neutrinos have a vanishing mass and mixing in vacuum. The positive oscillation indications in terrestrial and solar neutrino data are fully explained due to environmental effects that generate an effective mass and mixing in presence of baryonic matter. We perform an analysis of solar data [29, 30, 31, 32, 33, 34, 35, 36, 37], for one specific choice of parameters, obtaining an acceptable solution to the solar neutrino problem.

In Sec. II, we introduce the general theoretical framework which we will consider in this paper. In Sec. III we show how reactor and accelerator neutrino experiments test only the effective neutrino mass. In Sec. IV we discuss the neutrino flavour conversion probability in the Sun for this mechanism. In Sec. VI we address the problem for atmospheric neutrinos. Finally, in Sec. VII, we discuss our results and summarize our conclusions.

## II. MAVAN'S MECHANISM AND PARAMETERIZATION

In a previous work [23] we found limits for the product of the effective neutrino-scalar and matter-scalar Yukawa couplings described in [19]. But an assumption about the adiabaticity of the transition was made, and with this assumption we found that the new physics evoked always plays the role of a sub-leading effect compared to the standard oscillation scenario.

The aim of this work is to try to find if there is at least one combination of parameters for new physics that could lead to an acceptable solution to the neutrino oscillation data where such new physics is more than a sub-leading effect. This implies that non-adiabatic effects would be present in solar neutrino evolution.

A particular case from this oscillation plus MaVaN's scenario would be the extreme opposite from the one investigated previously, *i.e.* the situation where MaVaN's physics is the main flavour conversion mechanism in terrestrial experiments. This is the scenario we investigate in this paper, where the new physics generates the neutrino mass for all terrestrial

indications for neutrino oscillations, and is present in all three neutrino families.

The mass matrix in flavour basis has the form:

$$M = U_M \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix} U_M^{-1} + U_v \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U_v^{-1} \quad (1)$$

where  $U_M$  is the mixing matrix and  $M_i$  are the mass eigenvalues related to MaVaN's effect, while  $U_v$  and  $m_i$  are the mixing matrix and mass eigenvalues in vacuum. We assume that the mixing angles due to MaVaN's effects are constants.

The environment effect is introduced as a dependence of the mass terms with the baryonic matter density with the following parameterization:

$$M_i = M_0 \tanh \left( \lambda_i \frac{\rho}{3 \text{ g/cm}^3} \right) \quad (2)$$

This parameterization was chosen to reproduce two features that we want our mass matrix to present:

1. a linear growth of mass with baryonic density for small values of this density. This is the behavior suggested in [19], assuming a small shift in the value of  $A$  with respect to its ground value.
2. a saturation of the environmental dependence of neutrino masses for large values of the baryonic density.

### III. REACTOR AND ACCELERATOR NEUTRINOS

If we assume a constant Earth density in the crust, the neutrino oscillation probabilities can be written analytically, with the same form of the the vacuum oscillation probabilities in the known mass-induced oscillation scenario. The standard mass and mixing angles are replaced by the effective mass and mixing angles in matter.

The positive indications for neutrino oscillation in Earth experiments [38, 39, 40, 41] tell us that the mixing angles that diagonalize this mass matrix in presence of Earth matter are:

$$\begin{aligned} \sin^2 2\theta_{23} &> 0.90 \quad (90\% \text{ C.L.}) \\ 0.4 &< \tan^2 \tilde{\theta}_{12} < 2.4 \quad (95\% \text{ C.L.}) \\ \sin^2 \tilde{2}\theta_{13} &< 0.1 \quad (90\% \text{ C.L.}) \end{aligned} \quad (3)$$

with the following mass squared differences:

$$\Delta \tilde{m}_{21}^2 \approx 7.6 \times 10^{-5} \text{ eV}^2 \quad ; \quad \Delta \tilde{m}_{31}^2 \approx 2.5 \times 10^{-3} \text{ eV}^2 \quad (4)$$

We want to investigate the scenario where the MaVaN's are the leading neutrino flavour conversion mechanism, so we assume that the vacuum mass eigenvalues are very small,

$\Delta m_{ij}^2 \ll 10^{-5} \text{ eV}^2$ , and all the flavour conversion is induced by the first term in right-hand side of Eq. 1.

Assuming a constant Earth crust density of  $\rho \sim 3 \text{ g/cm}^3$ , we tune our parameters in eq. 2 in order to reproduce the above  $\Delta \tilde{m}^2$ . This can be achieved for instance with the following choice:

$$\begin{aligned} \lambda_1 = 0 \quad ; \quad \lambda_2 = 0.18 \quad ; \quad \lambda_3 = 10 \\ M_0 = 5 \times 10^{-2} \text{ eV} \end{aligned} \quad (5)$$

With this parameterization the mass of the second family is close to a linear regime for the baryonic densities present at Earth, and for the crust density of  $\rho \sim 3 \text{ g/cm}^3$  we obtain  $\Delta \tilde{m}_{21}^2 = 7.9 \times 10^5 \text{ eV}^2$ , with a  $\rho^2$  dependence. For the atmospheric neutrino scale, the third mass eigenvalue is already saturated for the Earth crust density, leading to  $\Delta \tilde{m}_{32}^2 = 2.4 \times 10^3 \text{ eV}^2$ .

Choosing a convenient set of mixing angles, the oscillation of terrestrial neutrino experiments are satisfactorily explained by this parameterization.

#### IV. SOLAR NEUTRINOS

For solar neutrinos, besides the environment effect in the mass matrix, the standard matter interaction term will have an important role in neutrino conversion. The evolution matrix has the form:

$$i \frac{d}{dr} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[ \frac{1}{2E_\nu} M M^t + \begin{pmatrix} V_{CC}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad (6)$$

where  $M$  is the mass matrix of Eq. 1.

The standard interaction term would be negligible at solar neutrino production point for the energies of interest, indicating that the mixing matrix that diagonalizes the full evolution matrix would be the same in KamLAND and in the center of the Sun.

However, the mass term decreases faster than the standard interaction one as the neutrino travels towards the Sun surface, and these terms become of the same order around  $r_{Sun} \sim 0.8$  for typical solar neutrino energies. The mixing matrix would start to feel the modifications due to the standard interaction term  $V_{CC}$  and the vacuum mass terms as the neutrino approaches the surface of the Sun.

In Fig. 1 and Fig. 2 we can see the behavior of the eigenvalues of the evolution matrix and the mixing angles inside the sun, for energies of 10 MeV and 1 MeV, respectively. We have chosen for the vacuum parameters:

$$\begin{aligned} \theta_{13} = \theta_{23} = \theta_{12} = 0 \\ \Delta m_{21}^2 = 10^{-9} \text{ eV}^2 \quad ; \quad \Delta m_{32}^2 = 2 \times 10^{-9} \text{ eV}^2 \end{aligned} \quad (7)$$

In this scenario the third family would assume a constant value for its mass eigenstate around  $r \sim 0.6$ , while the second family would achieve the saturation value around  $r \sim 0.2$ , corresponding to baryonic densities of 0.4 and 30 g/cm<sup>3</sup>, respectively.

We can estimate the survival neutrino probability assuming that the transition is adiabatic until very close to the solar surface, and then the neutrino assumes the vacuum values of mass and mixing in an extremely non-adiabatic transition. The electron neutrino is created as an admixture of the two first mass eigenstates,

$$|\nu_e\rangle = \cos\theta_{KL}|\nu_1\rangle + \sin\theta_{KL}|\nu_2\rangle$$

where I wrote  $\theta_{KL}$  to make it explicit that the mixing angle measured in KamLAND is the same at Sun's interior,  $\theta_{KL} = \tilde{\theta}_{12}(r=0)$ . For  $E = 10$  MeV (fig 1) the mixing angle increases as the neutrino travels towards the Sun's surface due to  $V_{CC}$ , and at  $r \sim 0.95$  the mixing angle achieves the value  $\sin\tilde{\theta}_{12} = 1$ . Assuming this change to be adiabatic, the probability to have an electron neutrino at this point can be estimated to be:

$$P_{ee} = \sin^2\theta_{KL}\sin^2\tilde{\theta}_{12} + \cos^2\theta_{KL}\cos^2\tilde{\theta}_{12} = \sin^2\theta_{KL} \sim 0.3 \quad ,$$

and the conversion probabilities are:

$$P_{e\mu} = P_{e\tau} \sim 0.35$$

Since in an extremely non-adiabatic transition there is no conversion between flavour eigenstates, these are the conversion probabilities right outside the Sun. Assuming no mixing in vacuum, these probabilities correspond to the probabilities measured at Earth, and to the admixture of mass eigenstates that evolve from Sun to Earth.

The Earth regeneration is expected to be small in this scenario. The key feature is that the vacuum mixing angle  $\theta_{23}$  was made very small, while  $\tilde{\theta}_{23}$  is maximum close to solar border. So the extremely non-adiabatic transition from the outer parts of the Sun to the vacuum leads to a strong production of the mass eigenstate  $\nu_3$ . And since we have  $P_{ee} \sim P_{e\mu} \sim P_{e\tau} \sim 1/3$  the non-adiabatic transition to vacuum will lead to an equipartition of  $\nu_1$ ,  $\nu_2$  and  $\nu_3$ , and then no regeneration effect at Earth. So the absence of Earth regeneration in Super-Kamiokande and SNO is a direct consequence of the probability  $P_{ee} \sim 1/3$  inside the Sun.

Doing the same analysis for  $E = 1$  MeV, we can estimate the probabilities the same way, with the difference that the mixing angle just before the non-adiabatic transition to vacuum is not changed by standard interaction term. Then:

$$P_{ee} = \sin^4\theta_{KL} + \cos^4\theta_{KL} \sim 0.6$$

and

$$P_{e\mu} = P_{e\tau} \sim 0.2$$

Again these probabilities correspond to the probabilities at Earth if we assume non-adiabatic transitions and no vacuum mixing. For these energies we would expect a strong

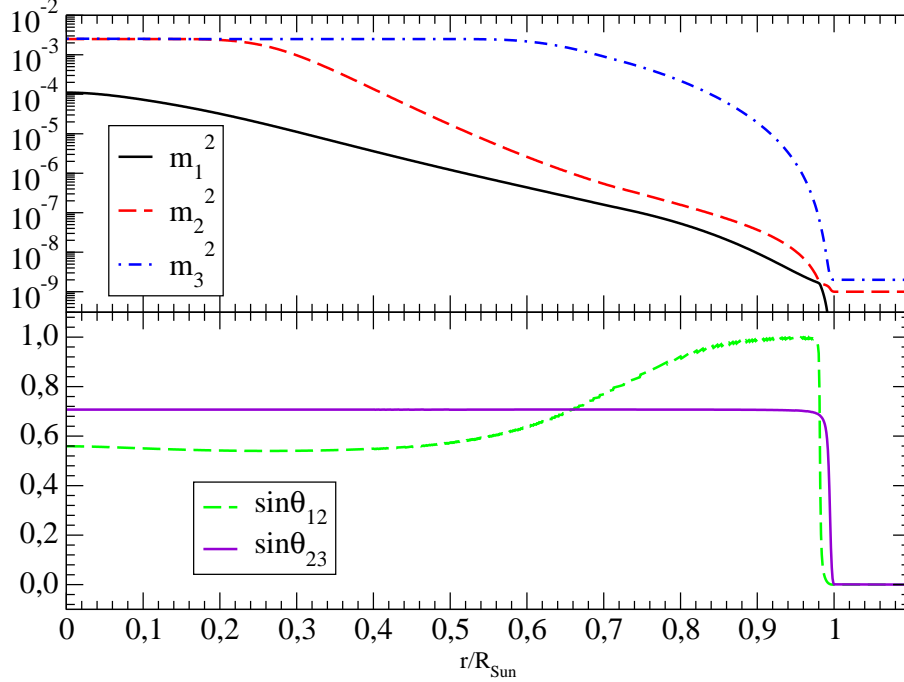


FIG. 1: Mass eigenvalues and mixing angles for  $E_\nu = 10$  MeV.

effect due to interaction with Earth matter, but since  $P_{\nu_1} > P_{\nu_2}$ , the Earth effect would lead to a stronger conversion at night.

We numerically calculated the solar neutrino survival probability for the values of  $\tan^2 \tilde{\theta}_{12} = 0.4$ ,  $\tan \tilde{\theta}_{23} = 1$  and  $\tilde{\theta}_{13} = 0$  for the mixing angles. For the vacuum parameters, we assumed very small values for the mass eigenvalues and mixing angles.

The result of our calculation can be seen in Fig. 3. All energy dependence present in the probability comes from non-adiabatic effects in neutrino evolution close to the border of the Sun.

The probability obtained reproduces two main ingredients of the desired conversion to explain the solar neutrino data: a higher survival probability for low energy neutrinos and a small regeneration effect for high energy neutrinos. Besides, there is one interesting feature of this mechanism that is the positive day-night asymmetry for low energy solar neutrinos, which could be tested by Borexino. Also, this would lead to a negative winter-summer asymmetry in low energy solar neutrino experiments due to the difference of day and night duration in winter and summer. GNO recently reported a winter-summer asymmetry of  $\Delta(W - S) = -7.6 \pm 8.4$  SNU ( $\sim -11\%$ ) in their full data analysis. The expected value from the  $1/d^2$  modulation only is  $+2.3$  SNU ( $+3.3\%$ ), slightly  $1\sigma$  above GNO results.

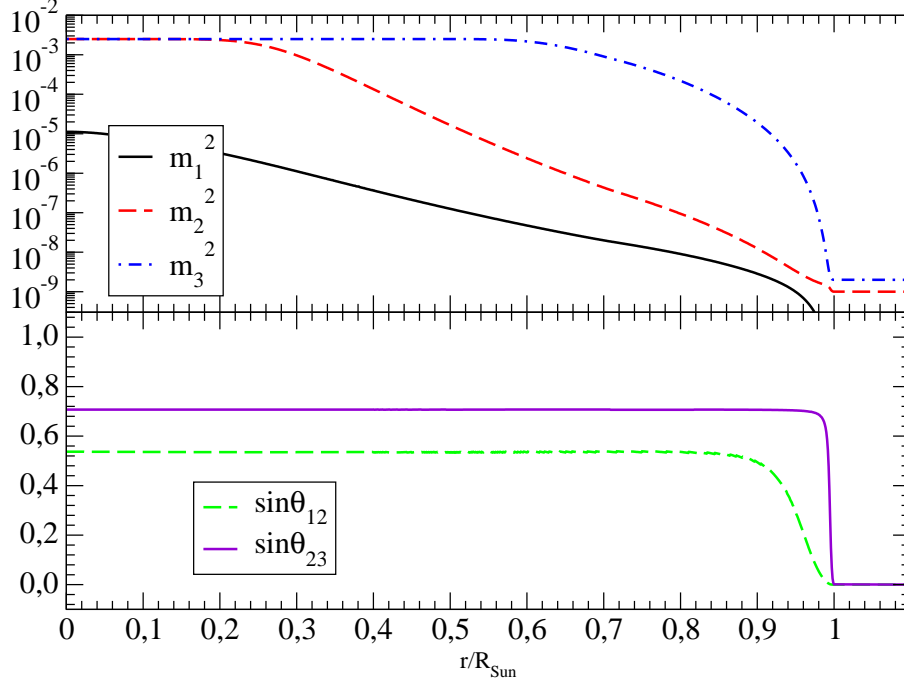


FIG. 2: Mass eigenvalues and mixing angles for  $E_\nu = 1$  MeV

## V. NUMERICAL ANALYSIS

We present in this section the results of the numerical analysis of solar data for the parameterization discussed in the previous section. Details of our solar neutrino analysis have been described in previous papers [23, 42, 43]. We use the solar fluxes from Bahcall and Serenelli (2005) [44]. In comparison with previous works, we include the new SNO data [36], and included latest Borexino results [37]. Besides, Gallex/GNO results were split by winter and summer data. The solar neutrino data includes a total of 124 data points:

- 1 data point for Homestake results [29].
- 1 data point for SAGE results [30].
- 2 data points for Gallex/GNO results, for winter and summer [31, 32].
- 44 data points for Super-Kamiokande zenithal/spectral bins [33].
- 34 data points for SNO, phase 1 [34].
- 38 points for SNO, phase 2 [35].
- 1 point for Borexino data [37]
- 3 points for SNO, phase 3 [36]

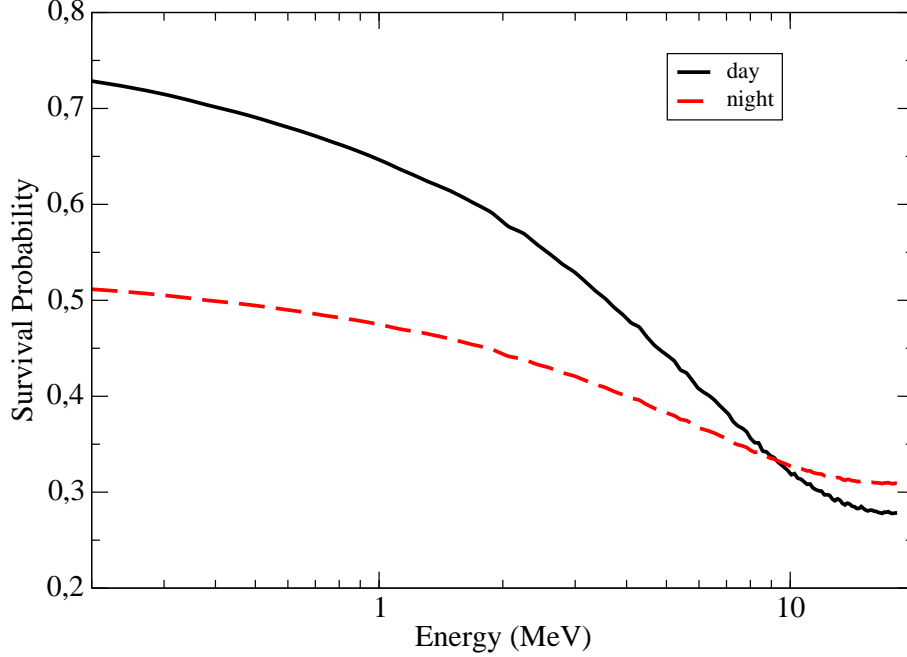


FIG. 3: Solar neutrinos survival probability during the day (solid line) and the night (dashed line).

We obtain a viable solution to solar neutrino problem with the following parameters.

$$\tan^2 \theta_{KL} = 0.4 \quad \Delta m_{KL}^2 = 7.9 \times 10^{-5} \text{ eV}^2 ,$$

with a  $\chi^2 = 118.9$ . For comparison, our standard analysis provides a  $\chi^2 = 113$  for the same parameters values.

As mentioned before, a clear signature of this mechanism would be the day-night asymmetry for low energy neutrinos. For the point specified above, the Berilium line neutrinos would have:

$$P_{day} = 0.62 \quad P_{night} = 0.45 ,$$

leading to a day-night asymmetry in Borexino of  $A_{DN} = 30\%$ .

For the winter-summer asymmetry in low-energy experiments, this same point would predict:

$$R_{summer} = 68.0 \quad R_{winter} = 68.7 ,$$

leading to a winter-summer asymmetry of  $A_{WS} = +1\%$ . MSW prediction for such asymmetry at the b.f.p. is around  $A_{WS} = +4\%$ .

## VI. ATMOSPHERIC NEUTRINOS

In this scenario, atmospheric neutrinos would oscillate through an almost constant  $\Delta m^2$  inside the Earth, with a small decrease in its value in Earth's core due to the rise of the second mass scale with higher densities. The possibility of environmental effects on atmospheric



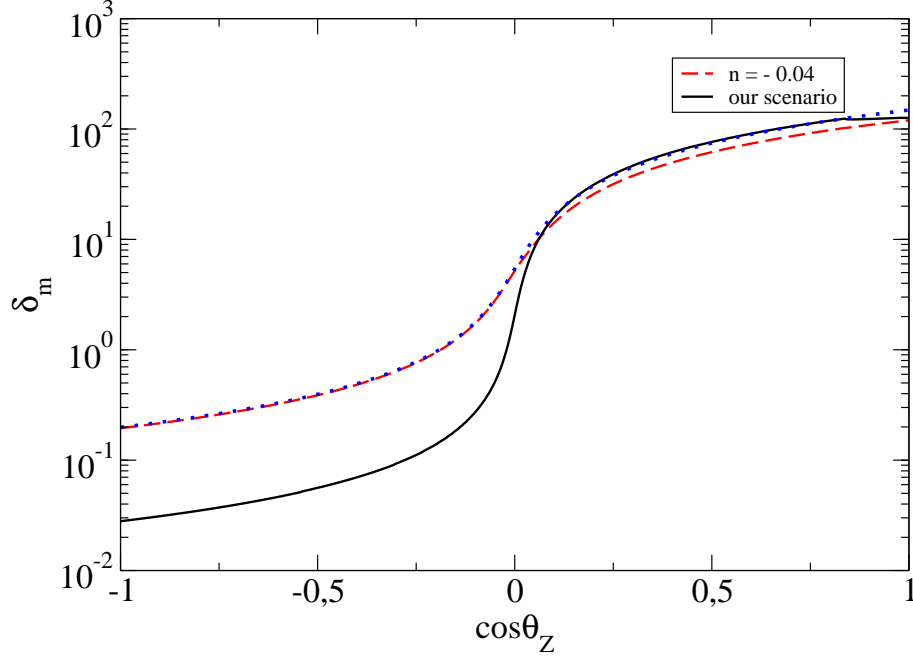


FIG. 4: Number of oscillation lengths for atmospheric neutrinos for our scenario (continuous line), standard oscillation (dotted line) and previous parametrization on MaVaN's mechanism (dashed line).

neutrinos were analyzed [27] with other parameterization of the matter-dependence. In that work the following choice:

$$\Delta m_{eff}^2 = (1.95 \times 10^{-3}) \left( \frac{\rho_e}{\rho_0} \right)^{-0.04} \text{ eV}^2$$

led to an acceptable solution to the SK atmospheric neutrino data. Here  $\rho_e$  is the electron neutrino density and  $\rho_0 = 6.02 \times 10^{23} \text{ cm}^{-3}$ .

To compare our proposal with the one above, we plot in fig. 4 the number of oscillation lengths covered by a typical atmospheric neutrino, given by:

$$\delta_m = \frac{\int \Delta m_{eff}^2 dL}{1 \text{ GeV}} .$$

The dashed line represents the choice in [27], and the solid line the  $\delta_m$  obtained with our parameterization. Also presented in dotted line is the standard oscillation scenario with constant  $\Delta m^2 = 2.3 \times 10^{-3} \text{ eV}^2$ .

We can see that the main difference between our choice of parameterization and the standard scenario happens for down-going neutrinos, with intermediate energy,  $E_\nu \sim \text{few GeV}$ . There are some indications that these neutrinos are already oscillating, which would decrease the concordance of this model with atmospheric neutrino oscillation data. But due to the very similar agreement with standard oscillation in a large range of atmospheric

neutrino flux parameters, we believe that an overall fit would give a viable solution also to atmospheric neutrinos. A detailed numerical analysis would be necessary to verify this issue.

## VII. CONCLUSIONS

We present in this article a possibility where all flavour conversion on neutrinos can come from environmental effects. The standard oscillation scenario is still the most elegant theoretical framework that explains neutrino flavour conversion, not only due to the excellent numerical fit to all oscillation data, but also due to the success of the research program that predicted, for instance, the correct signal at KamLAND based on one possible solution to the solar neutrino problem. The model proposed here explains such concordance *a posteriori*. However, it also make some very particular predictions on future experiments, that can be verified in the close future. Borexino experiment is already taking data, and very soon can test a possible day-night asymmetry for low energy solar neutrinos. The author does not know any other model that predicts such asymmetry, making this prediction an unique signature of MaVaN's as the mechanism on neutrino flavour conversion.

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[http://www.awa.tohoku.ac.jp/KamLAND/chi2map\\_3rdresult/chi2map.html](http://www.awa.tohoku.ac.jp/KamLAND/chi2map_3rdresult/chi2map.html)
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